Multivariate Volatility Impulse Response Analysis of GFC News Events

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Abstract: This paper applies the Hafner and Herwartz (2006) approach to the analysis of multivariate GARCH models using volatility impulse response analysis. The data set features ten years of daily returns series for the New York Stock Exchange Index and the FTSE 100 index from the London Stock Exchange, from 3 January 2005 to 31 January 2015. This period captures both the Global Financial Crisis (GFC) and the subsequent European Sovereign Debt Crisis (ESDC). The attraction of the Hafner and Herwartz approach is that it involves a novel application of the concept of impulse response functions, tracing the effects of independent shocks on volatility through time, while avoiding typical orthogonalization and ordering problems. Volatility impulse response functions (VIRF) provide information about the impact of independent shocks on volatility. Hafner and Herwartz's VIRF extends a framework provided by Koop et al. (1996) for the analysis of impulse responses. This approach is novel because it explores the effects of shocks to the conditional variance, as opposed to the conditional mean. Hafner and Herwartz use the fact that GARCH models can be viewed as being linear in the squares, and that multivariate GARCH models are known to have a VARMA representation with non-Gaussian errors. They use this particular structure to calculate conditional expectations of volatility analytically in their VIRF analysis. A Jordan decomposition of Σ_t is used to obtain independent and identically defined (iid) innovations. A general issue in the approach is the choice of baseline volatilities. VIRF is defined as the expectation of volatility conditional on an initial shock and on history, minus the baseline expectation that conditions on history. This makes the process endogenous, but the choice of the baseline shock within the data set makes a difference. We explore the impact of three different shocks, the first marking the onset of the GFC, which we date as 9 August 2007 (GFC1). This began with the seizure in the banking system precipitated by BNP Paribas announcing that it was ceasing activity in three hedge funds that specialised in US mortgage debt. It took a year for the financial crisis to come to a head, but it did so on 15 September 2008, when the US government allowed the investment bank Lehman Brothers to go bankrupt (GFC2). The third shock is 9 May 2010, which marked the point at which the focus of concern switched from the private sector to the public sector. A further contribution of this paper is the inclusion of leverage, or asymmetric effects. Our modelling is undertaken in the context of a multivariate GARCH model featuring pre-whitened return series, which are then analysed using both BEKK and diagonal BEKK (DBEKK) models with the t-distribution. A key result is that the impact of negative shocks is larger, in terms of the effects on variances and covariances, but shorter in duration, in this case a difference between three and six months, in the context of our particular return series.

Keywords: Volatility Impulse Response Functions (VIRF), BEKK, DBEKK, Asymmetry, GFC, ESDC.

JEL: C22, C32, C58, G32.

1. INTRODUCTION

The similarities between GARCH and VARMA-type models provide a foundation for the approach to generalize impulse response analysis, as introduced by Sims (1980), to the analysis of shocks in financial volatility. Previous alternative approaches in the literature have been made towards tracing the impact of various types of shocks through time (see, for example, Koop et al. (1996), Engle and Ng, (1993), Gallant et al. (1993), and Lin (1997)). Koop et al. (1996) defined generalized impulse response functions for the conditional expectation using the mean of the response vector conditional on history and a current shock, as compared with a baseline that conditions only on historical innovations.

Hafner and Herwartz's (2006) Volatility Impulse Response Functions (VIRFs) extend the generalized impulse response functions framework provided by Koop et al. (1996). Their approach is novel in that VIRF explores the conditional variance rather than the conditional mean. Given that GARCH models can be viewed as being linear in the squared innovations, and that multivariate GARCH models are known to have a VARMA representation with non-Gaussian errors, Hafner and Hewartz (2006) adopt this particular structure to calculate conditional expectations of volatility analytically in their VIRF analysis.

In our Generalized VIRF (GVIRF), we consider three major news events which act as shocks to the volatility of our two series. The onset of the GFC, which we date as 9 August 2007 (GFC1), began with the seizure in the banking system precipitated by BNP Paribas announcing that it was ceasing activity in three hedge funds that specialised in US mortgage debt. It took one year for the financial crisis to come to a head, but it did so on 15 September 2008 when the US government allowed the investment bank Lehman Brothers to go bankrupt (GFC2). The date 9 May 2010 marked the point at which the focus of concern switched from the private sector to the public sector. By the time the IMF and the European Union announced they would provide financial help to Greece, the issue was no longer the solvency of banks but the solvency of governments, and this marks the onset of the European Sovereign Debt Crisis (ESDC).

The remainder of the paper is as follows. In Section 2 the research methods and data are discussed, including volatility impulse response functions, multivariate GARCH models, the regularity conditions for BEKK and diagonal BEKK (DBEKK) models, the triangular, Hadamard and full BEKK models, and diagonal and scalar BEKK models. The empirical results are discussed in Section 3, and some concluding remarks are given in Section 4.

2. RESEARCH METHODS AND DATA

Hafner and Herwartz (2006) develop their model by letting ε_t denote an *N*-dimensional random vector, so that:

$$\varepsilon_t = P_t \xi_t , \tag{1}$$

where $P_t P_t^{'} = \sum_t$ and ξ_t denotes an *iid* random vector of dimension *N*, with independent components, mean zero and identity covariance matrix. Hafner and Herwartz assume that $\sum_t$ is measurable with respect to the information set available at time *t-1*, F_{t-1}. Equation (1) implies that $E\lfloor \varepsilon_t | F_{t-1} \rfloor = 0$, and $Var\lfloor \varepsilon_t | F_{t-1} \rfloor = \sum_t$. They note that ε_t could be the error of a VARMA process. If ε_t is a multivariate GARCH process, then equation (1) may be called a strong GARCH model, according to Drost and Nijman (1993). This is convenient because it permits the modelling of news events as appearing in the *iid* innovation, ξ_t. They identify ξ_t by assuming that P_t is a lower triangular matrix, which permits the use of a Choleski decomposition of $\sum_t$. They also use the fact that independent news can often be identified by means of a Jordan decomposition, which will permit identification when the innovation vector is non-normal.

Hafner and Herwartz adopt a multivariate GARCH(*p*,*q*) model framework, given by:

$$vech(\sum_t) = c + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-i}\varepsilon_{t-i}^{'}) + \sum_{j=1}^{p} B_j vech(\sum_{t-i}), \tag{2}$$

and use the BEKK model of Baba et al. (1985) and Engle and Kroner (1995), which is a special case of equation (2), and is specified as:

$$\sum_t = C_0 C_0^{'} + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ki}^{'} \varepsilon_{t-i} \varepsilon_{t-i}^{t} A_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} G_{ki}^{'} \sum_{t-i} G_{ki}. \tag{3}$$

In equation (3), C_0 is a lower triangular matrix, and A_{ki} and G_{ki} are $N \times N$ parameter matrices.

2.1 Volatility Impulse Response Functions

Hafner and Herwartz (2006) proceed by assuming that, at time *t*, some independent news is reflected in ξ_0, and it is not specified whether the news is good or bad. The conditional covariance matrix, $\sum_t$, is a function of the innovations, $\xi_1, \ldots, \xi_{t-1}$, the original shock, ξ_0, and $\sum_0$. Hafner and Herwartz define VIRF as the expectation of volatility conditional on an initial shock and on history, minus the baseline expectation that only conditions on history, as given in the following:

$$V_t(\xi_0) = E\left[vech\left(\sum_t\right) \middle| \xi_0, F_{-1}\right] - E\left[vech\left(\sum_t\right) \middle| F_{-1}\right] \tag{4}$$

In equation (4), $V_t(\xi_0)$ is an N^*-dimensional vector.

Hafner and Herwartz consider a VARMA representation of a multivariate GARCH(*p*,*q*) model in order to find an explicit expression for $V_t(\xi_0)$, and define $\eta_t = vech(\varepsilon_t \varepsilon_t')$. They define the multivariate GARCH(*p*,*q*) model as a VARMA(max(*p*,*q*), *p*) model:

$$\eta_t = \omega + \sum_{i=1}^{\max(p,q)} (A_i + B_i)\eta_{t-i} - \sum_{j=1}^{p} B_j u_{t-j} + u_t, \tag{5}$$

where $u_t = \eta_t - vech\left(\sum_t\right)$ is a white noise vector. From equation (5), Hafner and Herwartz derive the VMA(∞) specification, as follows:

$$\eta_t = vech\left(\sum\right) + \sum_{i=0}^{\infty} \phi_i u_{t-i}, \tag{6}$$

where the $N^* \times N^*$ matrices ϕ_i can be determined recursively. The general expression for VIRF is:

$$V_t(\xi_0) = \phi_t D_N^{+} (\sum_0^{1/2} \otimes \sum_0^{1/2}) D_N vech(\xi_0 \xi_0^{'} - I_N). \quad (7)$$

Hafner and Herwartz (2006) consider a variety of specifications for the baseline shock. The behaviour implied by equation (7) is different from traditional impulse response analysis. In (7), the impulse is an even, not odd, function of the shock, it is not linear in the shock, and the VIRF depends on the history of the process, although this is via the volatility state at the time the shock occurs. The decay or persistence is given by the moving average matrices, ϕ_t, which is similar to traditional impulse response analysis.

Further complications arise from the choice of baseline because no natural baseline exists for ε_0^0 in VIRF, as any given baseline deviates from the average volatility state. For example, a zero baseline would represent the lowest volatility state and volatility forecasts would increase from this baseline. After discussing various alternatives, Hafner and Herwartz (2006) adopt the definition given in equation (4). In their original analysis of exchange rates, Hafner and Herwartz examine the impact of particular historical shocks that occur in their sample, as well as considering random shocks for their estimated model.

In an empirical analysis of US and UK indices,we consider the onset of the GFC, which we date as 9 August 2007 (GFC1), then the date when the financial crisis came to a head, 15 September 2008, when the US government allowed the investment bank Lehman Brothers to go bankrupt (GFC2). The date 9 May 2010 marked the point at which the focus of concern switched from the private sector to the public sector, and this marks the onset of the European Sovereign Debt Crisis (ESDC). We also consider random shocks in the empirical analysis.

2.2 Multivariate GARCH Models

The analysis in the paper features applications of both the BEKK and Diagonal BEKK (DBEKK) models. The BEKK model was introduced by Baba et al. (1985) and Engle and Kroner (1995). In the case of a model with single lags, the BEKK recursion is:

$$H_t = CC^{'} + A^{'} u_{t-1} u_{t-1}^{'} A + B^{'} H_{t-1} B, \quad (8)$$

where ***H*** is a matrix of the covariances, and ***C***, ***A*** and ***B*** are the coefficient matrices. The expression above is written in *vech* format to generate the VIRFs, as shown below:

$$vec(H_t) = vec(CC^{'}) + (A^{'} \otimes A^{'})vec(u_{t-1}u_{t-1}^{'}) + (B^{'} \otimes B^{'})vec(H_{t-1}). \qquad (9)$$

However, a drawback of using the BEKK model is that there are no regularity conditions or statistical properties for full BEKK, as discussed in the next subsection. Chang et al. (2015) discuss stochastic processes for univariate and multivariate conditional volatility models, and the following subsections 2.3-2.5 draw closely on their analysis.

2.3 Regularity Conditions for BEKK and DBEKK

The original multivariate extension of univariate GARCH is given in Baba et al. (1985) and Engle and Kroner (1995), while a consideration of leverage effects and the multivariate extension of univariate GJR is given in McAleer et al. (2009). The asymmetry conditions for multivariate GJR are given in the VARMA-AGARCH model of McAleer et al. (2009). Leverage has typically been presented for individual equations only, as defined by Black (1976) for univariate processes using arguments based on the debt-to-equity ratio.

In order to establish volatility spillovers in a multivariate framework, it is useful to define the multivariate extension of the relationship between the returns shocks and the standardized residuals, that is:

$$\eta_t = \varepsilon_t / \sqrt{h_t},$$

where h_t denotes univariate conditional volatility. A multivariate extension of an equation for the conditional mean of financial returns can be written as:

$$y_t = E(y_t \mid I_{t-1}) + \varepsilon_t,$$

if it is assumed that the three components are $m \times 1$ vectors, where *m* is the number of financial assets. The multivariate definition of the relationship between ε_t and η_t is given as:

$$\varepsilon_t = D_t^{1/2}\eta_t \quad , \tag{10}$$

where $D_t = diag(h_{1t}, h_{2t}, ... h_{mt})$ is a diagonal matrix comprising the univariate conditional volatilities. Define the conditional covariance matrix of ε_t as Q_t. As the $m \times 1$ vector, η_t, is assumed to be *iid* for all m elements, the conditional correlation matrix of η_t, which is equivalent to the conditional correlation matrix of η_t, is given by Γ_t. Therefore, the conditional expectation of (10) is defined as:

$$Q_t = D_t^{1/2}\Gamma_t D_t^{1/2} \quad . \tag{11}$$

Equivalently, the conditional correlation matrix, Γ_t, can be defined as:

$$\Gamma_t = D_t^{-1/2} Q_t D_t^{-1/2} . \tag{12}$$

Equation (11) is useful if a model of Γ_t is available for purposes of estimating Q_t, whereas equation (12) is useful if a model of Q_t is available for purposes of estimating Γ_t.

Both equations (11) and (12) are instructive for a discussion of asymptotic properties. As the elements of D_t are consistent and asymptotically normal, the consistency of Q_t in equation (11) depends on consistent estimation of Γ_t, whereas the consistency of Γ_t in equation (12) depends on consistent estimation of Q_t. As both Q_t and Γ_t are products of matrices, neither the QMLE of Q_t or Γ_t will be asymptotically normal based on the definitions given in equations (11) and (12).

2.4 Triangular, Hadamard and Full BEKK

Without actually deriving the model from an appropriate stochastic process, Baba et al. (1985) and Engle and Kroner (1995) considered the full BEKK model, as well as the special cases of triangular and Hadamard (element-by-element multiplication) BEKK models. The specification of the multivariate model is the same as the specification in equation (8), namely:

$$H_t = CC' + A'u_{t-1}u_{t-1}'A + B'H_{t-1}B, \qquad (13)$$

except that A and B are full, Hadamard or triangular matrices.

Although estimation of the full, Hadamard and triangular BEKK models is available in some standard econometric and statistical software packages, it is not clear how the likelihood functions might be determined. Moreover, the so-called "curse of dimensionality", whereby the number of parameters to be estimated is excessively large, makes convergence of any estimation algorithm somewhat problematic.

Jeantheau (1998) showed that the QMLE of the parameters of the full BEKK model is consistent under a multivariate log-moment condition, while Comte and Lieberman (2003) showed that the QMLE are asymptotically normal under the assumption of the existence of eighth moments. Specifically, the multivariate log-moment conditions are difficult to verify when the matrices *A* and *B* are neither diagonal nor scalar matrices, and the eighth moment condition cannot be verified for a full BEKK model. Therefore, there are as yet no verifiable asymptotic properties of the full, Hadamard or triangular BEKK models.

2.5 Diagonal and Scalar BEKK

Consider a vector random coefficient autoregressive process of order one:

$$\varepsilon_t = \Phi_t \varepsilon_{t-1} + \eta_t \qquad (14)$$

where

ε_t and η_t are $m \times 1$ vectors, and Φ_t is an $m \times m$ matrix of random coefficients, and

$\Phi_t \sim iid(0, A)$,

$\eta_t \sim iid(0, QQ')$.

Technically, a vectorization of a full (that is, non-diagonal or non-scalar) matrix *A* to *vec A* can have dimension as high as $m^2 \times m^2$, whereas the half-vectorization of a symmetric matrix *A* to *vech A* can have dimension as low as $m(m+1)/2 \times m(m+1)/2$.

In a case where A is either a diagonal matrix or the special case of a scalar matrix, $A = aI_m$, McAleer et al. (2008) showed that the multivariate extension of GARCH(1,1) from equation (14), incorporating an infinite geometric lag in terms of the returns shocks, is given as the diagonal BEKK (DBEKK) or scalar BEKK model, namely:

$$Q_t = QQ + A\varepsilon_{t-1}\varepsilon_{t-1}^{'}A^{'} + BQ_{t-1}B^{'} \; , \qquad (15)$$

where *A* and *B* are both either diagonal or scalar matrices.

McAleer et al. (2008) showed that the QMLE of the parameters of the diagonal or scalar BEKK models were consistent and asymptotically normal, so that standard statistical inference on testing hypotheses is valid. Moreover, as Q_t in equation (15) can be estimated consistently, Γ_t in equation (12) can also be estimated consistently.

Given the above considerations, we present the results of both full BEKK and DBEKK in the empirical analysis that follows. We can be confident about the statistical properties of DBEKK when it is used to calculate VIRFs, and the important consideration is whether the two methods and their associated VIRFs, have the same implications for our results. If they point to the same conclusions, we can have more confidence in the results.

3. EMPIRICAL RESULTS

Summary statistics for the two index return series for the period 3 January 2005 to 31 December 2014, giving a total of 2608 valid observations, are shown in Table 1. Both the NYSE and the FTSE return series display excess kurtosis and are negatively skewed. The time series plots of the index values are shown in Figure 1.

Table 2 provides tests of skewness, kurtosis and whether the return series for the two index series are normally distributed. The Jarque-Bera (JB) test rejects normality at any standard level of significance. For this reason, the Student t distribution is used in the subsequent analysis. We filter the return series through an AR(1) process before proceeding to use the subsequent residuals in a multivariate BEKK analysis to generate the VIRF, as in Hafner and Herwartz (2006).

Table 3 shows the results of the application of the filters, and Table 4 gives the diagnostics for the residuals. The application of the AR(1) model appears to whiten the residuals, and the Ljung-Box Q statistics for serial correlation suggest that correlation is not a problem. The Jarque-Bera (JB) test strongly rejects normality for the shocks, so we conduct the subsequent analysis using the t-distribution.

3.1 Results from BEKK analysis

Table 4 shows the results of the application of the BEKK model. We can forecast the volatility and correlations for the two series using the BEKK model. We forecast for 100 days at the end of the time series and use a window of 400 daily observations to fit the model. The results are shown in Figure 2. The recent experience of relatively high volatilities cause the increase in the two forecast volatilities, while the correlation tends towards the mean over the sub-sample.

Plots of the VIRFs are shown in Figure 3, Panels A and B. The VIRF impulse responses for 9 August 2007, as shown in Panel A, use the variance at that point in time as the baseline. The initial response for the NYSE is scaled at just under 10000. When this is compared to the impulse response of the FTSE in the UK, the response is even larger at just over 10000. These have been computed using a baseline of the estimated volatility state, so they are excess over the predicted covariance. They can be contrasted with the impact of the EU debt crisis on 5 May 2010, in which the NYSE initial response is just over 1500, while the FTSE response at the same point in time is nearly 2000, suggesting that, as might be expected, the EU debt crisis had a larger impact in London than it had in New York.

These shocks have been predicted using a baseline of zero. The 2007 shocks take a period of about 6 months to work through, while the 2010 shocks take a longer period of 8-9 months, but this may well reflect the choice of a lower baseline. The covariances show a dramatic spike in response to both shocks but remain higher for longer, in relation to the 2010 shock, possibly in response to the choice of baseline, as mentioned above. Thus, the choice of baseline remains a key issue in the implementation of VIRF analysis.

Panel B of Figure 3 contrasts the 15 September 2008 GFC impact with the 5 May 2010 EU debt crisis once again, and the choice of baselines mirrors that in Panel A. The impact of the shock in 2008, at the height of the GFC, is relatively higher than previously, in both New York and London. On the NYSE it approaches 25000, while on the FTSE it is even higher, approaching 40000, and the shocks in both markets take longer to die out than they did in 2007, taking 9 months to return to equilibrium. The covariance approaches 20000 and remains at high levels for 6-7 months. The 5 May 2010 graphs are the same as in Panel A, and are included for the purpose of a direct comparison.

Given that we are considering VIRF in the context of stock market indices, it seems appropriate to consider asymmetry effects via the introduction of the separate consideration of the impact of negative shocks. The estimates of the BEKK and asymmetric BEKK-t models are shown in Tables 5 and 7, and the eigenvalues from BEKK-t and asymmetric BEKK-t are given in Tables 6 and 9, respectively (for the sake of brevity, only the multivariate GARCH and asymmetric terms are reported in the tables). The analysis is broadly similar as described above.

Figure 4 shows the VIRF (for the sake of brevity only September 2008 and May 2010 are considered). The key difference in the results, when compared to the previous analysis, is that the VIRFs are larger and of shorter duration. For example, the NYSE variance increases to 8000 and the FTSE variance increases to 15,000 in September 2008. The duration of the response for both 2008 and 2010 is reduced to 3 months for both the variances and covariances.

However, in Section 2.3 in this paper noted that we can be confident about the statistical properties of DBEKK when it is used to calculate VIRFs, which is not the case for full BEKK. The key finding is whether the two methods and their associated VIRFs have the same implications for the empirical results. If the empirical results lead to the same conclusions, we can have greater confidence in the empirical results. In Section 3.2 we present the empirical results and VIRFs from a diagonal BEKK (DBEKK) analysis.

3.2 Results from DBEKK

The DBEKK model has valid statistical properties and regularity conditions, so we can be confident in the empirical results. It has to be borne in mind that DBEKK has fewer parameters, so its VIRFs are simpler than are those for full BEKK. We estimate DBEKK using the same procedure as discussed previously, and use a t-distribution and include asymmetry.

The asymmetric DBEKK model estimated using a t-distribution (DBEKK-t) is much better behaved, as can be seen in Table 8. All the coefficients apart from one that are shown in Table 5 are significant. The eigenvalues shown in Table 9 are stable, given that all are less than one.

Figure 5 shows the impulse responses generated by the asymmetric DBEKK model estimated using a t distribution (DBEKK-t). The results in Panel A reflect the fact that the 9 August 2007 VIRF has a baseline calculated on the shock at that point in time, while the 15 September 2008 shock has a baseline of zero. The results are consistent with the previous BEKK estimates in that the asymmetric DBEKK model produces negative shocks that last for only 3 months in duration. The 2008 shocks again are larger in LFTSERET than on NYSERET.

Panel B in Figure 5 is constructed in a similar manner. The 9 August 2007 VIRF is calculated on the shock at that point in time, while the 15 September 2008 shock is calculated using a zero baseline. Consistent with the previous results, the shocks have a three-month duration, and their relative sizes are the same as previously calculated, revealing that both the BEKK and DBEKK results are entirely consistent.

In order to complete the analysis, we also calculate a DBEKK model without asymmetries and present the results in Tables 10-11 and in Figure 6. All the coefficients for the DBEKK model, without asymmetries, as shown in Table 10, are highly significant. The eigenvalues, as shown in Table 11, are closer to one than for the DBEKK model with asymmetries, as reported in Table 6, suggesting that the standard BEKK model is less stable.

In Figure 6, for purposes of comparison, we depict the VIRFs for the GFC2 period and the Euro debt crisis. The VIRFs in Figure 6 are consistent with the previous analysis using the full BEKK model without asymmetries. The impact of the 2008 shock is larger in London than in New York, using the shock at that point in time as a baseline. A similar pattern is observed in the 2010 Euro-debt shock. Once again, we observe, ignoring the asymmetries, the duration of the shock is much longer, and now extends to eighteen months in all figures before equilibrium is re-established. This is more than double the durations of the VIRFs recorded for the full BEKK model without asymmetries, but the relative durations remain consistent.

4. CONCLUSION

In this paper we have applied the Hafner and Herwartz (2006) Volatility Impulse Response Function (VIRF) analysis to ten years of daily return series from the New York Stock Exchange Index, and the London Stock Exchange FTSE 100 index, for the period 3 January 2005 to 31 January 2015. An attractive feature of VIRF analysis of the effects of shocks on volatility through time is that the shocks are treated as endogenous.

However, we also note that the choice of the baseline for the shock makes a considerable difference. A useful contribution of this paper is to consider asymmetric effects, which are well documented in the empirical analysis of stock markets (see, for example, Engle and Ng (1993)). We showed that the impacts of negative shocks are larger, but of shorter duration, than those implied by a symmetric treatment of shocks.

Our empirical analysis is based on application of the full BEKK model, for which no verifiable asymptotic properties exist, as well as the diagonal BEKK (DBEKK) model, which is not so constrained. The empirical results our consistent and suggest that the inclusion of asymmetries is important when VIRF analysis is applied to stock market data. It was found that the responses to negative shocks are deeper and of shorter duration than the responses to positive shocks. The empirical results of both the BEKK and DBEKK models are strongly consistent with each other.

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Table 1

Summary Statistics for 2005-01-03 - 2014-12-31 (2608 valid observations)

NYSERET (2608 valid observations)

Mean	Median	Minimum	Maximum
0.000154204	0.000431926	-0.102321	0.115258
Std. Dev.	C.V.	Skewness	Ex. kurtosis
0.0133989	86.8909	-0.417694	10.8634
5% Perc.	95% Perc.	IQ range	Missing obs.
-0.0202854	0.0179030	0.0103402	0

Summary Statistics for 2005-01-03 - 2014-12-31 (2608 valid observations)

FTSERET

Mean	Median	Minimum	Maximum
3.92100e-005	0.000475224	-0.105381	0.122189
Std. Dev.	C.V.	Skewness	Ex. kurtosis
0.0148037	377.549	-0.110113	9.87695
5% Perc.	95% Perc.	IQ range	Missing obs.
-0.0227705	0.0205110	0.0132403	0

Table 2

Tests of Skewness, Excess Kurtosis, and Normality

NYSERET(*100)			
Skewness	-0.417934	Signif Level (Sk=0)	0
Kurtosis (excess)	10.886570	Signif Level (Ku=0)	0
Jarque-Bera	12954.814995	Signif Level (JB=0)	0
FTSERET(*100)			
Skewness	-0.110176	Signif Level (Sk=0)	0.021693
Kurtosis (excess)	9.898215	Signif Level (Ku=0)	0
Jarque-Bera	10651.855632	Signif Level (JB=0)	0

Table 3

AR(1) and preliminary GARCH(1,1) analysis of return series

NYSE			
Variables	**Coefficient**	**t-statistic**	**Significance**
Constant	0.054269041	3.39885	0
LNYSERET(1)	-0.050346740	-2.49472	0.013
GARCH(1,1)			
C	0.016988318	2.95313	0.003
A	0.093671095	6.40479	0
B	0.893694731	61.55474	0
FTSE			
Constant	4.7248e-004	2.35012	0.019
LFTSERET(1)	-0.0463	-2.27302	0.023
C	1.7113e-006	2.90809	0
A	0.0911	5.66440	0
B	0.9013	52.15142	0

Table 4

Residual diagnostics

ARCH-LM(1)	**JB**	**Q(10)**	**Q(20)**
LNYSERET			
8.476 (0.004)	472.482 (0.000)	9.000 (0.437)	23.055(0.235)
LFTSERET			
0.002 (0.967)	197.09 (0.000)	5.125 (0.823)	17.914(0.528)

Table 5

BEKK

Variable	Coefficient	Standard Error	t-statistic	Significance
Constant	0.094673045	0.015120103	6.26140	0
LNYSERET{1}	-0.252211378	0.018119393	-13.91942	0
Constant	0.077323881	0.019894664	3.88666	0
LFTSERET{1}	-0.168032092	0.016587251	-10.13020	0
C(1,1)	-0.097175963	0.044805916	-2.16882	0.03
C(2,1)	-0.264611585	0.034032404	-7.77528	0
C(2,2)	-0.000000180	0.149309283	-1.20715e-006	0.999
A(1,1)	0.021678144	0.041879070	0.51764	0.605
A(1,2)	-0.383455482	0.052098541	-7.36020	0
A(2,1)	-0.222393062	0.035195693	-6.31876	0
A(2,2)	-0.063023626	0.046314167	-1.36079	0.173
B(1,1)	1.202152703	0.015121227	79.50100	0
B(1,2)	0.450960714	0.027752985	16.24909	0
B(2,1)	-0.354541888	0.021500835	-16.48968	0
B(2,2)	0.591348452	0.024731239	23.91099	0
Shape	7.670707369	0.748939459	10.24209	0

Table 6

Eigenvalues from BEKK-t

0.98025	0	0.72696	-0.46101	0.72696	0.46101
Var	JB	p-value			
1	147.280	0			
2	69.556	0			
All	216.836	0			

Table 7

Asymmetric BEKK-t

Variable	Coefficient	Standard Error	t-statistic	Significance
A(1,1)	-0.022753722	0.060798967	-0.37425	0.708
A(1,2)	-0.405700847	0.065933722	-6.15316	0
A(2,1)	0.148631275	0.035519302	4.18452	0
A(2,2)	0.296233075	0.041308360	7.17126	0
B(1,1)	0.812855262	0.026787787	30.34425	0
B(1,2)	-0.151242974	0.031493570	-4.80234	0
B(2,1)	0.161414758	0.030535132	5.28620	0
B(2,2)	0.997063705	0.025611106	38.93091	0
D(1,1)	-0.469369500	0.036937131	-12.70725	0
D(1,2)	-0.393521072	0.089578341	-4.39304	0
D(2,1)	0.211373660	0.061407304	3.44216	0
D(2,2)	-0.083147397	0.085927903	-0.96764	0.333
Shape	8.904691765	0.951329821	9.36026	0

Table 8

Asymmetric DBEKK-t

Variable	Coefficient	Standard Error	t-statistic	Significance
Mean Model LNYSERET				
Constant	0.072214891	0.016514826	4.37273	0
LNYSERET(1)	-0.246671385	0.017309242	-14.25085	0
Mean Model LFTSERET				
Constant	0.051226153	0.019264661	2.65907	0.008
LFTSERET(1)	-0.129102063	0.016647036	-7.75526	0
C(1,1)	0.122517499	0.012861431	9.52596	0
C(2,1)	0.110032035	0.015744065	6.98879	0
C(2,2)	0.088019683	0.012074757	7.28956	0
A(1)	-0.024217524	0.033245856	-0.72844	0.466
A(2)	-0.150597648	0.029857611	-5.04386	0
B(1)	0.959878240	0.004026069	238.41572	0
B(2)	0.959775221	0.005034805	190.62807	0
D(1)	0.338891628	0.018669042	18.15260	0
D(2)	0.283093998	0.025964433	10.90315	0
Shape	7.623084667	0.738881477	10.31706	0

Table 9

Eigenvalues from Asymmetric BEKK-t

0.94383, 0	0.92489, 0	0.92193, 0
Var	**JB**	**p-value**
1	153.216	0
2	224.941	0
All	378.157	0

Table 10

DBEKK-t without Asymmetries

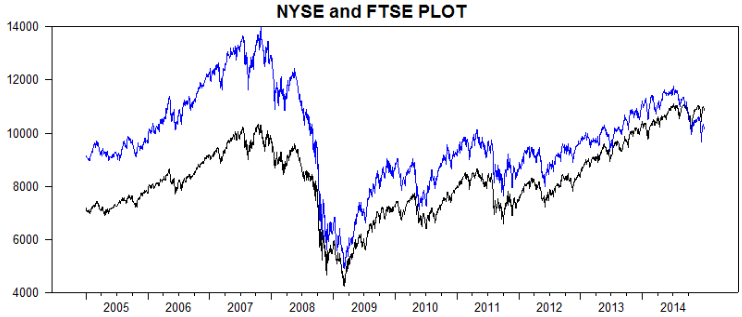
Variable	Coefficient	Standard Error	t-statistic	Significance
Mean Model LNYSERET				
Constant	0.090305522	0.015901813	5.67895	0
LNYSERET(1)	-0.251500344	0.017757663	-14.16292	0
Mean Model LFTSERET				
Constant	0.064511941	0.019540751	3.30141	0.001
LFTSERET(1)	-0.138112219	0.016239859	-8.50452	0
C(1,1)	0.120332752	0.014853367	8.10138	0
C(2,1)	0.079599176	0.013060471	6.09466	0
C(2,2)	0.092005900	0.013195478	6.97253	0
A(1)	0.281404331	0.016505582	17.04904	0
A(2)	0.243537494	0.016343016	14.90162	0
B(1)	0.954923410	0.005051244	189.04719	0
B(2)	0.966108091	0.004134165	233.68881	0
Shape	6.754575562	0.611797521	11.04054	0

Table 11

Eigenvalues from BEKK-t

0.99268, 0	0.99109, 0	0.99107, 0
Var	**JB**	**p-value**
1	159.968	0
2	240.138	0
All	400.106	0

Figure 1



Note: NYSE - Blue, FTSE – Black.

Figure 2

100 day forecasts based on BEKK

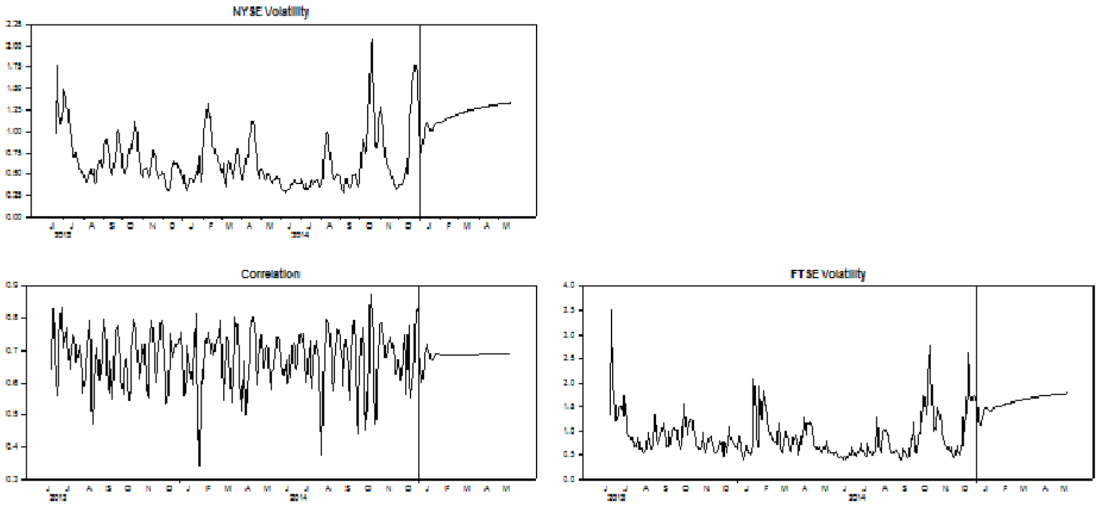
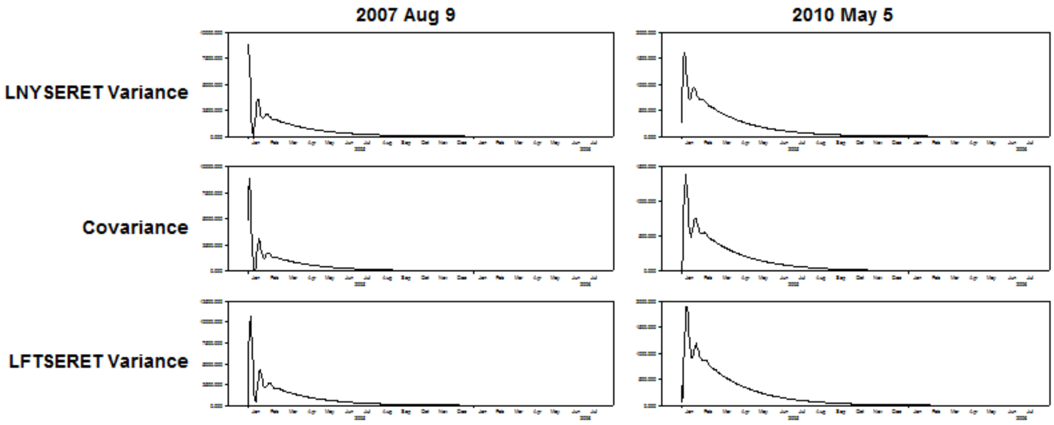


Figure 3

VIRF Panel A: Baselines 9 August 2007 and 5 May 2010



VIRF Panel B: Baselines 15 September 2008 and 5 May 2010

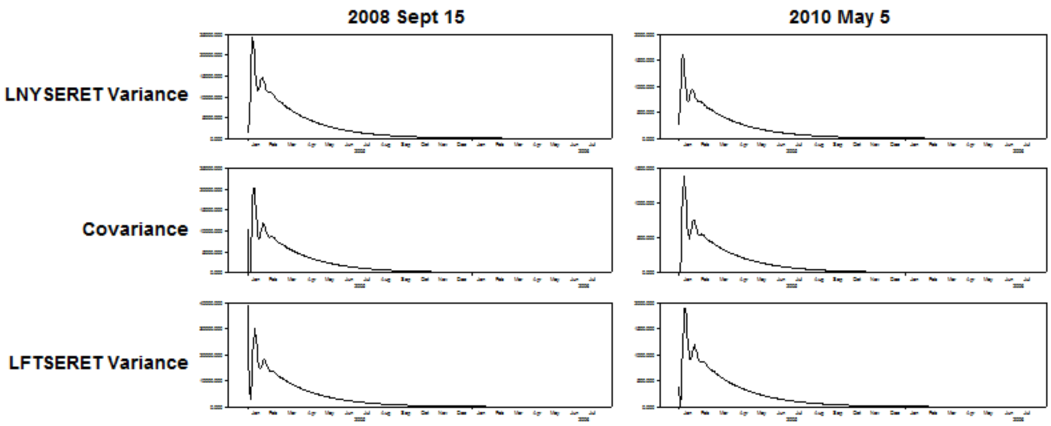


Figure 4

VIRF Asymmetric BEKK (responses to negative price movements)

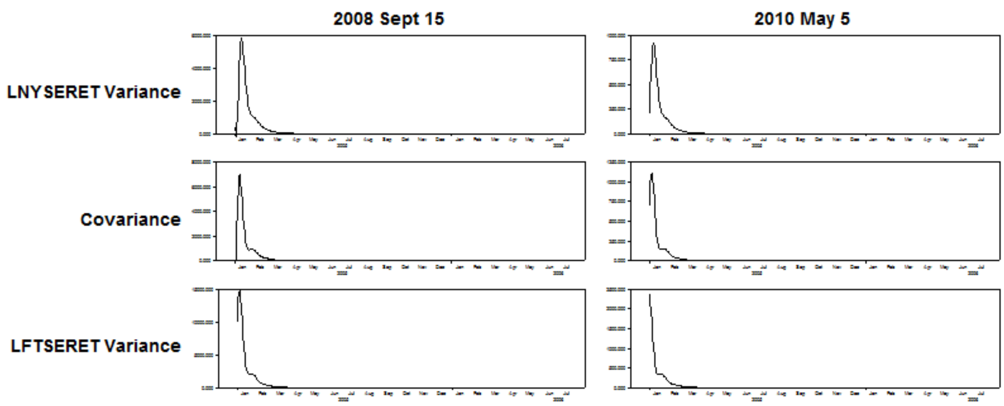
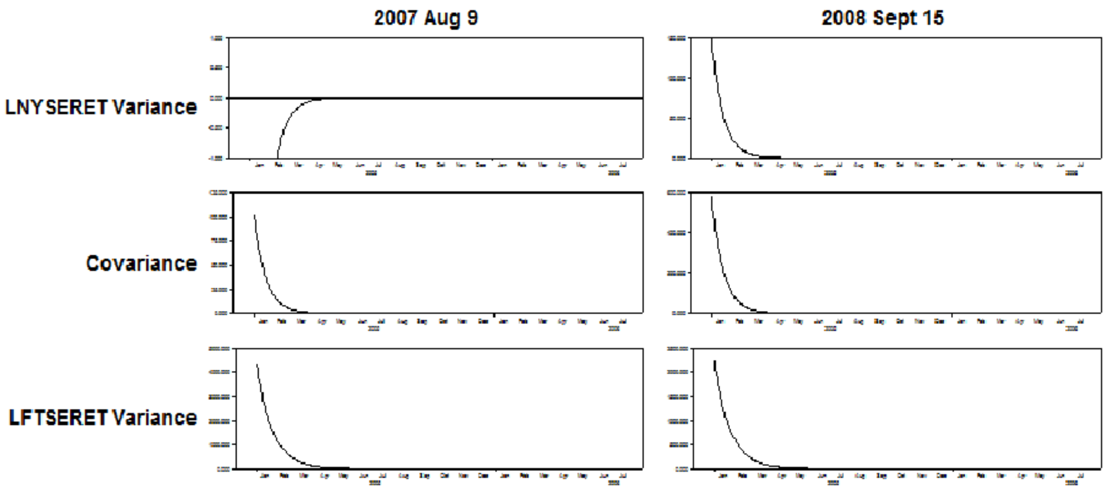


Figure 5

VIRF Asymmetric DBEKK-t

Panel A



Panel B

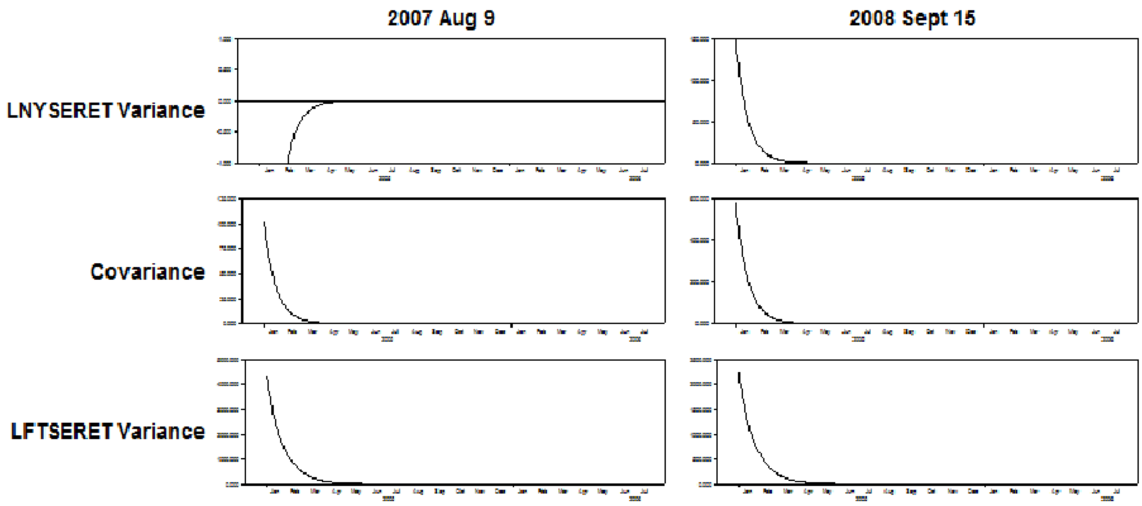


Figure 6

VIRF for GFC2 and Euro Debt crisis using DBEKK-t