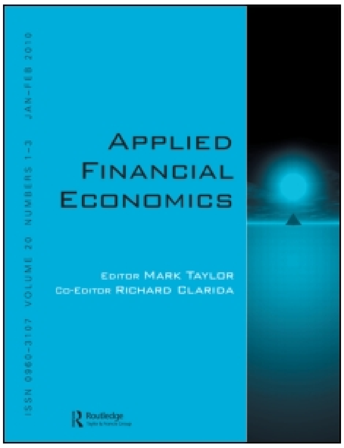

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Empirical performance of affine option pricing models: evidence from the Australian index options market

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Empirical performance of affine option pricing models: evidence from the Australian index options market

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This article investigates the performance of affine option pricing models in the context of the Australian Standard & Poor's (S&P)/Australian Stock Exchange (ASX) 200 index option market. This investigation is done through the implicit estimation of the risk neutral parameters of affine option pricing models using S&P/ASX 200 index options data between January 2001 and December 2006. In particular, Stochastic Volatility (SV) and jumps in both price and volatility are considered. Our research indicates that call options are best modelled with a process that includes SV and jumps in price and volatility, while put options are best modelled with a process that allows SV and jumps in price (but not in volatility). Under the assumption of near constant parameters through time a more parsimonious model is the best choice, with a plain SV model performing best for call options and a jump-diffusion or a SV model performing equally well for put options.

I. Introduction

Option markets have grown spectacularly over the past three decades throughout the world, and with this growth in the market came the growth in demand for accurate pricing methods. As noted by Bakshi *et al*. (1997), the sheer number of models, even a decade ago, was overwhelming. Choosing the best model for option pricing today, therefore, is a very difficult task. A model should be judged on a number of criteria, including fit to the data, internal consistency and analytic tractability. While most complicated models have had good success in improving fit and some success in achieving consistency, tractability has become a serious issue in option pricing (Garcia *et al*., 2005).

One particular class of models that have succeeded in also achieving analytic tractability is the class of affine Jump-Diffusion (JD) option pricing models. Models in this class may be defined with complex processes for the underlying price and option pricing is provided through the transform and inversion methods of Bakshi and Madan (2000) and Duffie *et al*. (2000). This results in semi-closed form formulae that require only univariate numeric integration of a well-behaved function. Models in this class

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include the Stochastic Volatility (SV) models stemming from the work of Heston (1993), the JD models beginning with Merton (1976) and Bates (1991) as well as combinations of the two such as in Bates (1996, 2000) and Bakshi *et al.* (1997). Some of the most complex models of recent times, with SV and jumps in both prices and volatility, such as Eraker *et al.* (2003) and Chernov (2007) are also members of this class.

The relative empirical desirability of some of these models for the US Standard & Poor's (S&P) 500 index options market has been studied by many researchers. Most of these advanced models, however, have not been studied in the Australian context. The double jump model considered in this article has, to the best of authors' knowledge, not been studied empirically in any context previously. The need for the investigation of these models is confirmed by Twite (1996) and Brown (1999), who look at the implicit volatility of S&P/Australian Stock Exchange (ASX) 200 index futures options. They each document consistent biases, concluding that there is a need for a more complex option pricing model more consistent with the underlying distribution.

The aim of this article, therefore, is to introduce a more complex class of models to the Australian context, and study their utility and the information they convey about the implicit underlying distribution. As such, the purpose of this article is two-fold. Firstly, it aims to test the empirical desirability of each of these models in the Australian context and look at the information about the implicit distribution provided. Secondly, it aims to study the internal consistency of the models used and provide evidence on possible misspecification.[1]

Our research reveals a number of interesting findings on the Australian index options market. We find that the distribution implicit in call options is substantially different to that of put options. It is difficult to determine, however, whether the difference is a true one or a reflection of the different structure of the call and put option data samples. We find that the best model for fitting call options is a SV with jumps in price and volatility while for put options the best model is a SV with jumps in price but not volatility. Under the assumption of near constant parameters through time a more parsimonious model is the best choice, with a plain SV model performing best for call options and a JD or SV model for put options. The choice, however, is shown to be a decision between significantly misspecified models through analysis of model internal consistency.

The remainder of this article is structured as follows. Section II reviews some of the relevant literature. Section III introduces and explains the data sources. Following this, Section IV formally introduces the models to be studied and discusses all relevant methodological issues. Section V presents the empirical results of the article while Section VI concludes.

II. Review of Relevant Literature

An option can be perfectly priced if we know the expected future value of the underlying and the appropriate discounting kernel. The Black–Scholes (BS) model finds the expected value of the underlying using a log-normal assumption for price changes. This approach, however, has been shown by many to produce a very poor fit empirically. Obvious biases, often characterized as smiles or skews in the implicit volatility of that model have been documented extensively.[2]

Options are not redundant securities as assumed by the BS model, but in fact span relevant risk factors.[3] This implies not only that the risk free rate cannot be used in discounting the expected value of the underlying, but also a model that accounts for these risk factors must be used in finding this expected value. Risk factors, such as volatility risk (Pan, 2002) and jump risk (Chernov, 2007) have been investigated and shown to be relevant as well as general un-parametrized risks (Ait-Sahalia and Duarte, 2002).

This article concentrates on parametric forms for volatility and jump risk in modelling the underlying since this provides more informative conclusions about the distribution of underlying price changes and is well supported empirically (Garcia *et al.*, 2005). Generally, the modelling of these risk factors is best done with fully stochastic specifications, often involving a further state variable for the level of volatility, such as models from the class of affine models studied in this article (Dumas *et al.*, 1998).

These stochastic models aim to capture the common empirical features of returns, such as time changing and mean reverting variance whose

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[1] This empirical testing has only recently become possible with the growth of the Australian index options market. At the beginning of 2001 (the start of this sample) there were on average about one to five liquidly traded index option series each day. This has grown to an average of about 35 at the end of the sample.

[2] See, for example, Rubinstein (1985, 1994) and Dumas *et al.* (1998).

[3] Buraschi and Jackwerth (2001) demonstrate this formally for the US.

innovations are correlated with price changes (Engle and Patton, 2001) and sudden, large, discontinuous changes in prices (Bates, 1991) and volatility (Naik, 1993). Models capable of addressing some or all of these features include the SV model of Heston (1993), the JD model of Merton (1976), the variance gamma model of Madan *et al.* (1998), the constant elasticity of variance model of Jones (2003) and the combined SV and jumps models of Bates (1996), Bakshi *et al.* (1997) and Duffie *et al.* (2000).

Option pricing using these complicated stochastic models can be very difficult to apply, especially in the case of the variance gamma and constant elasticity of variance models (Bakshi *et al.*, 2006). The SV and JD models, and their combinations, being part of the class of affine models, however, have semi-closed form solutions available due to the work of Bakshi and Madan (2000) and Duffie *et al.* (2000), extending Heston (1993) and Stein and Stein (1991). These solutions use transform analysis and inversions to reduce option pricing to simple univariate integration, which we address with a speedy method of nonadaptive quadrature.

The use of these solutions, however, requires knowing not only the level of the latent volatility state variable, but also the parameters that describe the model. Risk premiums associated with these newly defined risks must also be dealt with. These risk premiums mean that the objective parameters that describe the actual changes in the underlying (under the objective probability measure P) cannot be used to price options using the risk free rate.[4] There does exists an equivalent martingale measure, the risk neutral probability measure Q, under which the risk neutral parameters can be used to price options using the risk free rate for discounting. The risk neutral parameters, however, cannot be estimated from the time-series of returns due to the transformation to the risk neutral probability measure.[5]

Implicit parameter estimation is a way to recover the risk neutral parameters using option price data. Under the assumption of accurately measured option prices, an appropriate cross-section of option prices can be used to effectively invert an option pricing model and exactly recover the parameters of the underlying risk neutral returns distribution (Bates, 2003). Knowing these risk neutral parameters allows us to learn about an option pricing model and the implicit view an option market has of the risk neutral return distribution, as well as measure the fit and internal consistency of a model. This type of estimation is used here to study not only the option pricing models under consideration, but also the implicit distributions of call and put options in Australia.

This approach has been used by many researchers to study models from the class of affine option pricing models, such as Bakshi *et al.* (1997), Bates (1996, 2000) and Pan (2002). These models have had success in improving over the standard BS model, often used as the 'straw man' to measure improvement against (Bates, 2003). Bates (1996, 2000) and Bakshi *et al.* (1997) all find that SV offers the greatest first order improvement in fit, and that jumps in prices are also important. They do, however, find that the process is still significantly misspecified and call for the inclusion of jumps in volatility.[6]

The results regarding the utility of a jump in volatility process is mixed. Eraker *et al.* (2003) find that it is very relevant for fitting the time-series of returns, although Eraker (2004) finds that a simultaneous jump process where jumps in price and volatility are correlated and occur at the same time is not relevant in fitting options data (through implicit estimation). An independent jump in volatility process is used here, similar to that specified by Chernov (2007), although he does not test this process with regard to fit but rather uses it to investigate volatility forecasting. This article is the first study of such an independent jump in volatility model in the context of options data.

III. Data and Sample Statistics

Data

This article uses data on S&P/ASX 200 option prices, the underlying S&P/ASX 200 index levels and the Bank Bill Swap (BBSW) interest rate, a proxy for the risk free rate of return. The data cover the period from 1 January 2001 to 31 December 2005.

The daily close options data are provided by the ASX.[7] S&P/ASX 200 index options are European in style and cash-settled with quarterly expirations. They are generally available over a wide variety of

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[4] This means that time-series methods for extracting the parameters of such models from the returns distribution, such as those which are used in Andersen *et al.* (2002) and Eraker *et al.* (2003), are not particularly useful for option pricing.

[5] Pan (2002) attempts to characterize risk premiums by comparing the objective and risk neutral parameters of models similar to those studied here.

[6] This conclusion is reached even when a two factor volatility process, such as that used in Bates (2000) and Chernov *et al.* (2003) is tested.

[7] www.asx.com.au or +61 131279. We thank the Australian Stock Exchange for the provision of the data.

exercise prices and several maturities, though only a subset of these are traded on any given day.

The risk-free interest rate data are sourced from the Bloomberg data service. The BBSW Rates, as calculated by the Australian Financial Markets Association (AFMA),[8] for various maturities are the primary series. The BBSW however, does not cover the entire sample for all maturities. In the earlier part of the sample, the predecessor to the BBSW 1-year rate is used. We use 1, 2, 3, 4, 5 and 6 month rates and a 1 year rate. The interest rate to be used with each option is matched to the option's maturity by linearly interpolating the two nearest interest rate series.

The index levels are also obtained from the Bloomberg data service. The data are the daily close level of the index underlying S&P/ASX 200 index options, the S&P/ASX 200 index (ASX code 'XJO'). The data cover the entire sample period.

Since S&P/ASX 200 options are European style, put-call parity may be used to calculate put prices from the associated call price under the assumption of no transaction costs (Stoll, 1969).[9] We can use the put-call parity formula

$$P(\tau) = C(\tau) + Xe^{-r\tau} - S_0 \tag{1}$$

where $P(\tau)$ is the price of a put maturing in τ periods (years) and $C(\tau)$ is the price of a call maturing in τ periods (years). X is the exercise price of the option, S_0 is the initial index level and r is the risk free rate of return.

To prepare the data for estimation, a number of offending daily option prices are removed for varying reasons. Following Bakshi *et al.* (1997), all observations that do not satisfy the following minimum value arbitrage constraints are removed

$$C(\tau) \geq \max[0, S_0 - XB(\tau)], \tag{2}$$

$$P(\tau) \geq \max[0, XB(\tau) - S_0] \tag{3}$$

where $B(\tau)$ is the current price of a \$1 zero coupon bond with the same maturity as the option.

Imposing this condition results in the removal of about 8% of observations at this stage, much larger than the 1.3% removed in Bakshi *et al.* (1997). Due to much lower liquidity in the options market in Australia, it was expected that there would be a larger number of observations violating this condition in the Australian market compared to the US S&P 500 index options market used more commonly in studies. It should also be noted that the removal of the observations breaching Equations 2 and 3 alleviates the problem due to the fact that the options prices and index levels may not be synchronous.[10]

Certain S&P/ASX 200 index option series have also been excluded for various reasons. These include Low Exercise Price Options (LEPOs)[11] which behave more like a forward contract and the very short-term options (those with less than 5 days to maturity) which may also introduce bias.

It should be noted that for S&P/ASX 200 index options, the tick-size is one point (ASX, 2006b). This equates to about 1.5% of the average option price. As such, no problems with price-discreteness are expected.

Index performance during the sample period

Initial analysis of the S&P/ASX 200 index returns (log-differenced price levels) data yields immediate evidence of nonnormality. The skew and the excess kurtosis for the sample period are −0.55 and 7.4, respectively. The Jarque–Bera (Bera and Jarque, 1980) test for normality confirms the nonnormality of returns. We reject the hypothesis of normality with a *p*-value smaller than 1×10^{-6}.

Another common empirical feature that the option pricing models studied here aim to capture is time-changing volatility. The first graph in Fig. 1 shows the annualized level of S&P/ASX 200 index volatility throughout the sample period. Volatility is calculated using a 21 day window and we can see easily that it does not appear to be constant throughout the sample. Also notable is that although volatility changes over time, it appears to be roughly mean reverting, as observed by Engle and Patton (2001). The SV models, of the type to be used in this article, are ideally placed to capture these features.

One may also note several very sudden, large positive changes in volatility (about three peaks are easily seen) that occur from time to time, followed by reversion back to the mean. These jumps in volatility cause a clustering of large returns (such as those described by Eraker, 2004) and these clusters can be observed in the second graph of Fig. 1. Such behaviour is allowed for by the double jump model in this article.

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[8] See the website http://www.afma.com.au/for more information on the AFMA.

[9] This is necessary since all option pricing formulae used are specified for call options. This is the method suggested by Bates (1996, 2000) to price put options from the call option formulae.

[10] We note that the daily close index levels may not be the optimal ones to use. However, due to the lack of transaction data and this filtering, the remaining observations should be reasonable for the purpose of this study.

[11] See ASX (2006a) for more information on LEPOs.

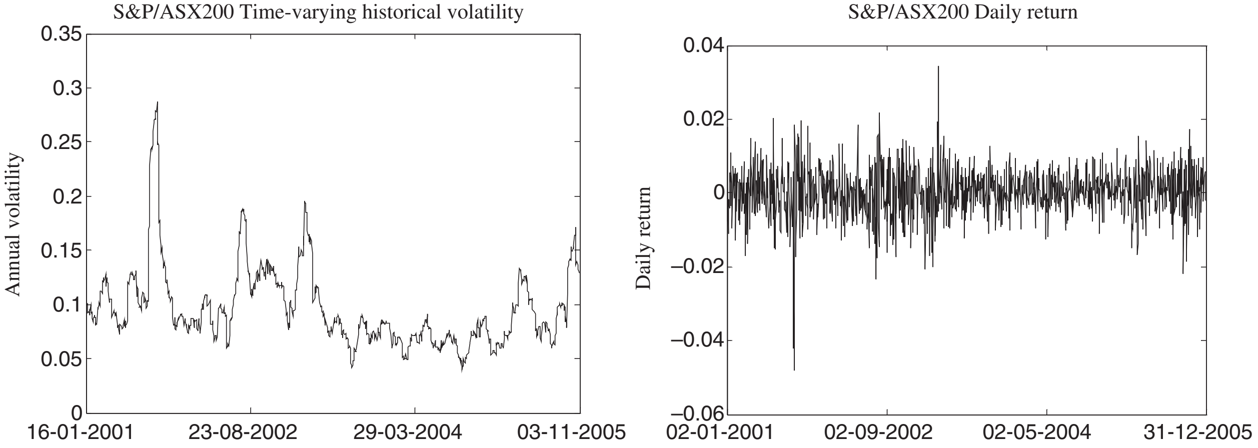

Fig. 1. Time-varying historical volatility and return
Notes: The first graph shows the volatility of the S&P/ASX 200 index log returns series, calculated over a 21 day (1 month) window centred on each day for which there is enough data to do so. Volatility levels are annualized based on a 250 trading days in a year. The second graph shows daily returns throughout the sample.

Sample statistics

Table 1 summarizes some of the properties of the sample call and put data. The data are divided into nine categories using moneyness and term to expiration. The most common options are medium-term and at-the-money. Long-term and In-The-Money (ITM) options appear least frequently. Put options tend to be much more likely to be Out-of The Money (OTM) than call options, although for both types of options the average moneyness is out of, rather than ITM. These differing characteristics of the sample of put and call data are relevant in discussions in later sections.

The average option maturity, daily volume and open interest are similar between calls and puts, though the number of traded series is greater for puts, indicating greater overall activity in this type of option. This allows the more complex models to be estimated on more days for put options than for call options. Average market prices for options are also reported and, as expected, value increases, on average, with term-to-expiration and moneyness.

IV. Models and Methodology

Option pricing models

The double jump (SVJJ – stochastic volatility with jumps in price and volatility model) model is the most complex of the models to be used in this article. This specification is derived from the general specification of Duffie *et al.* (2000). A double jump specification with simultaneous price and volatility jumps, also derived from the Duffie *et al.* (2000) framework, has been studied by Eraker *et al.* (2003) and Eraker (2004).[12] Our specification, with independent price and volatility jumps has not been studied previously in this manner.[13] We define the underlying price process in terms of risk neutral parameters as follows:

$$\frac{\mathrm{d}S(t)}{S(t)} = \left(r - \bar{\lambda}\bar{\mu}\right)\mathrm{d}t + \sqrt{V(t)}\mathrm{d}W_s(t) + J_s(t)\mathrm{d}q_s(t) \quad (4)$$

where $V(t)$ is itself stochastic and obeys the following process:

$$\mathrm{d}V(t) = \kappa[\bar{v} - V(t)]\mathrm{d}t + \sigma_v\sqrt{V(t)}\mathrm{d}W_v(t) + J_v(t)\mathrm{d}q_v(t) \quad (5)$$

J_s is modelled as

$$\ln[1 + J_s(t)] \sim N\left[\ln(1 + \mu_s) - \frac{\sigma_J^2}{2}, \sigma_J^2\right] \quad (6)$$

and J_v is modelled as

$$J_v(t) \sim \exp\left[\frac{1}{\mu_v}\right] \quad (7)$$

As shown by Duffie *et al.* (2000), under risk neutrality, the following must hold:

$$\bar{\lambda} = \lambda_s + \lambda_v \quad (8)$$

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[12] They study a model with a single jump process and simultaneous jumps in prices and volatility. They find that this type of double jump model does not improve the fit to options data.

[13] Chernov (2007) uses a similar independent jump setup to study risk premiums in volatility forecasting, but does not study options data.

Table 1. Sample properties of S&P/ASX 200 index options

Moneyness	*Maturity* < 60 (short)	60 ≤ *maturity* < 180 (medium)	*Maturity* ≥ 180 (long)	Total
Panel I: Call options				
Maturity (days)				
OTM	9.51 pts	27.37 pts	51.76 pts	29.85 pts
$M < 0.97$	(744)	(2745)	(1001)	(4490)
ATM	42.36 pts	79.62 pts	126.12 pts	67.51 pts
$0.97 < M < 1.03$	(3181)	(4071)	(553)	(7805)
ITM	287.31 pts	231.04 pts	260.85 pts	261.71 pts
$M > 1.03$	(276)	(230)	(17)	(523)
Total	52.64 pts	64.00 pts	80.20 pts	62.26 pts
	(4201)	(7046)	(1571)	(12818)
Sample averages				
S/X	Maturity (days)	Volume	Open interest	Series traded per day
0.98	97.55	76.17 contracts	833.10 contracts	10.56 series
Panel II: Put options				
Maturity (days)				
OTM	11.99 pts	30.69 pts	58.08 pts	32.30 pts
$M > 1.03$	(1953)	(4610)	(1827)	(8390)
ATM	47.16 pts	81.62 pts	122.06 pts	72.20 pts
$0.97 < M < 1.03$	(3148)	(4019)	(822)	(7989)
ITM	223.03 pts	242.20 pts	334.69 pts	247.11 pts
$M < 0.97$	(475)	(809)	(176)	(1460)
Totals	49.82 pts	70.51 pts	93.93 pts	67.75 pts
	(5576)	(9438)	(2852)	(17839)
Sample averages				
S/X	Maturity (days)	Volume	Open interest	Series traded per day
1.04	105.88	67.75 contracts	818.59 contracts	14.78

Notes: This table reports the average prices and number of observations (in brackets) that fall into each category. In addition, some sample averages are also reported. The sample covers the years 2001–2005. S denotes the index level and X the exercise price. OTM, ATM and ITM denote Out-of-The-Money, At-The-Money and In-The-Money options, respectively. Moneyness is defined as $M = S/X$. Panel I shows the statistics for call options, while Panel II shows those for put options.

and

$$\bar{\mu} = \frac{1}{\lambda}\left[\lambda_s \exp\left(\mu_s + \frac{\sigma_J^2}{2}\right) + \lambda_v\right] - 1 \qquad (9)$$

The Weiner processes W_s and W_v are correlated with coefficient ρ allowing for the so-called leverage effect (Chernov, 2007). The Poisson jump counters q_s and q_v, with jump intensities λ_s and λ_v are independent of each other, allowing jumps in prices and volatility to occur at different times.[14] Jumps in price are log-normally distributed with expected $J_s = \mu_s$ and variance σ_J^2 and jumps in volatility are exponentially distributed with expected $J_v = \mu_v$.

One should notice that the mean volatility jump size can be only positive (and hence all volatility jumps must be positive) otherwise the exponential distribution would not be defined. The process is defined in this way intentionally by Duffie *et al.* (2000). For a detailed interpretation of the jumps in price process, one may see Bates (1991), while for the SV process and its combination with jumps in price, one may see Bates (1996) and Bakshi *et al.* (1997).

All the other models studied are nested in this framework. To recover the SV with jumps in price model (SVJ), identical to that used in Bates (1996) and Bakshi *et al.* (1997) λ_v is set to zero. To recover the SV model, identical to that of Heston (1993), λ_s is also set to zero. The JD model of Merton (1976) and Bates (1991) is then found by setting $V(t)$ to a constant value and λ_s to a nonzero value. The BS model of Black and Scholes (1973) is further recovered by setting λ_s to zero.

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[14] Restricting the process to simultaneous jumps in price and volatility was explored by Eraker (2004) and found not to improve the fit to options data over a process with no jumps in volatility.

Empirical methodology

Implicit parameter estimation. Implicit parameter estimation is the method used to estimate the parameters of the option pricing models in this article. Garcia *et al.* (2005) suggest that this type of study of the risk neutral parameters is a 'more informative exercise' compared to the use of time-series methods and objective parameters. As found by Chernov and Ghysels (2000), using only options data is the preferred method for recovering risk neutral parameters, rather than using a combination of options data and underlying index data.

Recovering the risk neutral parameters implicitly involves selecting a basket of options, at least as many as the number of variables to be estimated, and selecting an objective function to measure fit. We follow many other studies that use this method and choose an objective function that minimizes the sum of squared absolute pricing errors,[15] i.e.

$$\min f(V(t),\Theta)=\sum_{i=1}^{n}\left[C_{mkt,i}-C_{mdl,i}(V(t),\Theta)\right]^2 \quad (10)$$

where $C_{mkt,i}$ is the market price of the *i*-th option in the basket, $C_{mdl,i}$ is the model price of the *i*-th option in the basket, $V(t)$ is the level of implicit volatility and Θ represents the parameter vector, whose length is determined by the model being estimated.

The minimization of the objective function (Equation 10) could be viewed as a nonlinear least squares problem and solved using some Nonlinear Least Squares (NLS) algorithm (Bates, 2000). We formulate the problem as the following NLS regression:

$$C_{mkt,i}=C_{mdl,i}(V(t),\Theta)+\varepsilon_i \quad (11)$$

For example, we need to estimate 10 parameters for the SVJJ model: $V(t)$ and $\Theta=[\kappa,\bar{v},\sigma_v,\rho,\lambda_s,\lambda_v,\mu_s,\mu_v,\sigma_J]$.

An important consideration before performing the NLS minimization in Equation 11 is the form of the errors, ε_i. As noted by Bates (2003), there is no established theory for the form of option pricing errors. Bates (1996, 2003) notes that these errors must be at least heteroskedastic with respect to exercise price and maturity and are generally contemporaneously correlated across different exercise price-maturity categories and serially correlated over time.[16]

Since the heteroskedasticity and correlation problems should theoretically only affect SEs of the daily estimates, we should not expect bias in the parameter and state variable estimates due to this.[17] We therefore proceed to use the NLS algorithm LSQNONLIN.M, part of the MATLAB package. This algorithm is based on the trust region method and requires the numeric evaluation of derivatives.[18]

It should be noted that using derivative based methods such as the NLS algorithm does not guarantee convergence to the global minimum, especially in problems of higher dimensionality, such as in the case of the more complex models studied in this article (Lange, 1998). In these cases, the minimum found can be dependent on starting values. These minimums will generally be good approximations to the global minimums if the starting values are good approximations to the global minimum parameters (Brandimarte, 2002). Starting values were chosen based on a combination of intuition obtained through extensive experimentation and the results of studies of similar models in the US. This was generally found to be acceptable, with alternate starting values tending to result in either convergence to the same point or larger errors.

As mentioned, this estimation is done for each day in the sample for which there are enough traded option series to create a large enough basket. Since the larger models require more options in their baskets, they tend to be estimated on fewer days of the sample than the smaller models. This creates a time-series of parameters of which we may take the mean/median over any time period to arrive at final parameter values for that period. We will generally use either the full sample (5 years) time period or yearly sub-periods.

Using the mean of the daily estimations would provide a good estimate of the true value of the parameter in a large sample where we would expect our sample of parameters values to converge to a normal distribution by the central limit theorem (assuming a stationary implicit distribution

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[15] Other studies that adopt an implicit parameter estimation method with this objective function include Bates (1991, 1996, 2000), Bakshi *et al.* (1997), Dumas *et al.* (1998), Madan *et al.* (1998) and Duffie *et al.* (2000).

[16] Bates (2000) suggests a whole sample Kalman filter methodology in combination with the nonlinear least squares methods of Engle and Mustafa (1992) to deal with these issues though this method is limited to estimating a single set of parameters over the entire sample.

[17] Bias should only be seen in the SEs due to the heteroskedasticity and correlation directly. It is however, obvious that these problems are manifest of some underlying misspecification that may cause bias, but this is a problem we must live with when using any option pricing model (Bakshi *et al.*, 1997; Bates, 2000).

[18] See More and Sorensen (1983) for further information on this method.

in option prices). Bakshi *et al.* (1997) claim to use 'basic histogram based inference' to confirm approximate normality for their data and find that using the mean (and its SE) is satisfactory.

In many cases in this article, however, the parameter estimates do not appear to converge to a normal distribution, often exhibiting large skew and high kurtosis. This lack of convergence is something that would be expected if the underlying implicit distribution in option prices is not stationary. Given that any perfectly specified model will be far too complex to be dealt with, the choice between parsimonious, but misspecified models means such problems as nonstationarity of some parameters will emerge (Bakshi *et al.*, 1997; Bates, 2000).

To deal with this nonnormality of distributions, we choose to use the median, rather than the mean, to summarize central tendency. This choice gives a much better estimate of the true 'average' parameter value through time. This is confirmed by analysis of option prices generated from median versus mean parameter values. Prices generated by the mean parameter values are generally far too high due to the positive skew of the distribution of some parameters (especially, the jump process related parameters).

V. Empirical Results

In addressing the questions of how to characterize the option implicit distribution in Australia and which model is best to use for pricing these options, we turn to two general approaches. Firstly, we analyse the fit of the models by comparing pricing error both using daily estimation and under the assumption of constant (slow-changing) parameters. This gives a good indication of which model is best able to give accurate option prices and hence fit the implicit distribution in options. Secondly, we study the implicit parameters of the models to understand the features of the implicit distribution in options and evaluate the usefulness of each option pricing model feature. This shows which features of these more complex models are relevant in the fitting of put and call options. We also look at the internal consistency of these parameters and study how in some cases they appear to change markedly through time.

Model fitness analysis

In analysing the five option pricing models (BS, JD, SV, SVJ and SVJJ) presented in Section IV, we invert each day's sets of option prices to obtain the implicit risk neutral parameters and volatility level for each different model. Each day has two sets of options, the first set contains all call options traded on the day and the second set contains all traded put options.

The median of the parameters and volatility level over all days under consideration is taken for each model and option-type pair and their 90% confidence intervals are calculated.[19] In judging fit, we will consider all days that the option's implicit parameters were successfully estimated in calculating the median. While this affords a bias to smaller models that are able to easily fit days with fewer options due to less variation in exercise prices and maturities, this is balanced by the larger models having more free parameters.[20] The results are reported in Table 2 which reveals that the order of ranking of the models, based on the Mean Squared Error (MSE) per option is not the same for both put and call options. This difference in ordering is the first evidence that there are different processes implicit in call and put options.

The ranking, from poorest fit to best fit for call options is: BS, JD, SVJ, SV and SVJJ. For put options, the ordering is different, being BS, JD, SV, SVJJ and SVJ. Overall, it appears that put option implicit distributions are best modelled by a process that allows firstly for stochastic volatility, and secondly, but also importantly, jumps in price. Jumps in volatility do not appear to be relevant.

This contrasts with call option implicit distributions, which are best modelled with a process that includes SV and jumps in prices and volatility, though the benefit of jumps in prices is unclear. These differences in fit underline the fundamental

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[19] This method for bootstrapping the confidence interval of the median is based on DeGroot and Schervish (2002) extended to use the Lunneborg method for application to possibly asymmetric distributions. It is also adapted to allow for the autocorrelation in parameter values. For a given sample of parameter values, we retake 1000 sub-samples (of the same size as the sample and with replacement) from this sample. This sampling is done in blocks of values that match the length of the lag structure in the parameter series. We then create a series of the medians of each of these sub-samples. The lower confidence interval is then twice the original sample median minus the $(1-\alpha/2)$-th percentile of this series and the upper confidence interval is twice the original sample median minus the $(\alpha/2)$-th percentile of this series. Generally, confidence intervals state here will be calculated with α set to 10%, giving a 90% confidence interval.

[20] Initial results for model fit were calculated using only the days on which every model was estimated, forcing smaller models to span the same variation in exercise prices and maturities of larger models. The results for this were qualitatively and quantitatively very similar and are available upon request.

Table 2. Whole-sample implicit risk neutral parameters

Model	BS	JD	SV	SVJ	SVJJ
Panel I: Call options					
Days	1182	1031	976	744	561
Imp. Vol.	8.447%	6.960%	9.472%	9.090%	10.295%
	8.156%	*6.516%*	8.813%	*8.470%*	*9.510%*
	8.758%	*7.353%*	10.126%	*9.715%*	*11.231%*
κ			5.295	5.765	4.818
			5.242	*5.673*	*4.523*
			5.337	*5.840*	*5.095*
$\sqrt{\bar{v}}$			7.021%	5.321%	6.446%
			6.649%	*4.850%*	*5.777%*
			7.307%	*5.774%*	*7.161%*
σ_v			15.316%	11.902%	29.312%
			13.879%	*10.769%*	*26.738%*
			16.750%	*13.050%*	*32.024%*
P			0.463	0.389	−0.397
			0.327	*0.219*	*−0.610*
			0.609	*0.564*	*−0.261*
λ_s		1.062		0.670	0.558
		1.034		*0.648*	*0.487*
		1.105		*0.688*	*0.646*
λ_v					0.610
					0.571
					0.645
μ_s		2.843%		2.908%	5.276%
		2.435%		*2.616%*	*4.541%*
		3.457%		*3.756%*	*6.233%*
μ_v					6.614%
					3.017%
					8.904%
σ_J		0.026%		0.129%	8.610%
		0.021%		*0.020%*	*7.349%*
		0.029%		*0.166%*	*9.804%*
MSE	36.893 pts	32.130 pts	9.952 pts	12.045 pts	9.131 pts
Panel II: Put options					
Days	1171	1075	1035	909	801
Imp. Vol.	15.301%	9.202%	13.958%	10.711%	12.305%
	14.901%	*8.847%*	13.379%	*10.401%*	*11.836%*
	15.629%	*9.506%*	14.463%	*11.212%*	*12.750%*
κ			3.820	3.256	3.167
			3.651	*2.862*	*2.961*
			3.979	*3.574*	*3.452*
$\sqrt{\bar{v}}$			22.917%	16.205%	6.293%
			22.357%	*15.409%*	*5.268%*
			23.370%	*17.029%*	*7.323%*
σ_v			141.775%	51.533%	65.803%
			135.643%	*46.765%*	*54.781%*
			150.883%	*55.409%*	*74.851%*
P			0.085	−0.229	−0.372
			0.059	*−0.280*	*−0.452*
			0.103	*−0.154*	*−0.318*
λ_s		0.574		0.446	0.646
		0.516		*0.380*	*0.524*
		0.637		*0.532*	*0.733*
λ_v					0.471
					0.435
					0.497
μ_s		−0.008%		5.236%	1.610%
		−0.713%		*4.173%*	*0.869%*

(*continued*)

Table 2. Continued

Model	BS	JD	SV	SVJ	SVJJ
		0.701%		*6.332%*	*2.356%*
μ_v					41.482%
					38.003%
					44.237%
σ_J		21.129%		16.659%	16.106%
		20.045%		*15.548%*	*11.780%*
		22.584%		*18.077%*	*21.177%*
MSE	89.138 pts	17.242 pts	11.112 pts	9.250 pts	10.533 pts

Notes: This table shows the median volatility level and risk neutral parameters, with their 90% confidence intervals in italics, for each model for both call options in Panel I and put options in Panel II. All days in which there were enough options to estimate a models are included in taking the median. The MSE for a model is calculated as the total (over all included sample days) sum squared error divided by the total (over all days) number of options used in the calculations. The parameters are: Imp. Vol = implicit volatility, κ = speed of reversion of volatility to long-run mean, $\sqrt{\bar{v}}$ = long-run mean volatility ($\bar{v}$ is long-run mean variance), σ_v = volatility of volatility, ρ = correlation between volatility and return changes, λ_s = jump intensity in the underlying price, λ_v = jump intensity in volatility, μ_s = expected value for price jump size, μ_v = expected value for volatility jump size and σ_J = volatility of price jumps.

differences in the implicit distributions of call and put options.

Under the assumption of constant (or slow-changing) parameters, we can recalculate the daily sum squared error per option using the yearly median of parameter values rather than the daily values. In this case the level of volatility is still estimated each day. Table 3 shows the MSE per option for each model for each data set. It is immediately apparent, due to the much higher errors, that all the models significantly violate the assumption of constant or near-constant parameter values.[21] By this measure, an SV model is best for call options, while either an SV or a JD model perform equally well for put options. Significant differences again exist between call and put options and there is evidence of over-fitting by the more complex models. We see that while the fit of all the models has decreased substantially, the decrease becomes worse as the number of parameters increases. The SVJJ model, the best performing model for call options and second best performing model for put options using daily estimation, is now the worst performing model for both data sets (performing even worse than the BS model).

While the implicit distributions of call and put options are clearly different, this is not positive evidence that call and put options behave differently. The characteristics of the data set are such that the average moneyness and maturity is different for call and put options. Due to this, any differences between put and call implicit distributions could be due to some biases of the models used that are related to these contract-specific variables.

Simple regression analysis of the errors of the models, similar to Bakshi *et al.* (1997), indicates exercise price and a maturity related biases for all models under consideration.[22] As such, we cannot distinguish between whether the implicit distributions of calls and puts differ or the models are significantly misspecified. Further references to the differences in implicit distributions should be read with this in mind.

Implicit parameters and internal consistency

Looking at the implicit parameters of a model allows us to characterize the implicit distribution in call and put options. This gives an indication of the features that are most useful in fitting option prices and suggests how best to model the implicit distribution, an important question to be answered.

Internal consistency refers to the consistency of the estimated parameters of a model with the model's assumptions. For example, if a model assumes constant parameters and these parameters change over time, it is not internally consistent. Internal inconsistency is a good indicator of misspecification in a model's assumptions.

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[21] Apart from the BS model as it has no parameters to be affected by being held constant.

[22] Results of this analysis are not presented here to save space and are available on request.

Table 3. Near-constant parameter option pricing model fit

	BS	JD	SV	SVJ	SVJJ
Calls – MSE	36.81 pts	35.40 pts	20.99 pts	24.82 pts	51.28 pts
Puts – MSE	88.97 pts	34.09 pts	34.62 pts	39.95 pts	90.56 pts

Notes: This table shows the MSE per option for each model for each data set (puts and calls). The MSE for a model is calculated as the total (over all days) sum squared error divided by the total (over all days) number of options used in the calculations.

Implicit volatilities. Looking at the difference in implicit volatilities between call and put options, we can see that implicit volatility is higher for put options for all models under consideration. Since the majority of options in the data set are OTM, this evidence that OTM call options are trading at much lower implicit volatilities than OTM put options suggests a serious implicit volatility skew (and hence a moneyness bias). The size of this skew, however, appears to be decreasing as more complex model features are added, indicating that these more complex models are better able to model this empirical feature.

Jumps in price parameters. Looking at call options, the jump size SD is low for all models and almost negligible for the JD and SVJ models, although the number of jumps per year is significant. Identified by Naik and Lee (1990) as the most important contributor of variance to the jump process, the jump size SD is so small that the jump process does not have much significance in determining option prices. This, combined with only a minor decrease in MSE, indicates that jumps are not useful in fitting the distribution implicit in call options.

For put options, it can be seen that in this case, the jump process appears to contribute considerably. The SD of the jump size is large in all cases, with significant but infrequent jump intensities (of around 0.5 jumps per year) indicating a significant contribution to the total variance of the models is due to the jump process and this is reflected in a much lower MSE.

SV parameters. Comparing the volatility of volatility between calls and puts, there is a huge discrepancy in size. The values for puts as high as 142% are up to five to ten times larger than those for calls and seemingly unreasonably high. Reports in other studies find values in the range of about 30% (for puts and calls in Bates, (1996)) to 75% (for puts and calls in Bates, (2000)). These numbers for puts seem far too high to be consistent with the time-series of implicit volatilities, suggests that there is significant implicit excess kurtosis in put options than these models cannot capture.

The addition of jumps in price reduces the volatility of volatility for put options by a factor of around three, while the introduction of jumps in price and jumps in volatility both reduce the long-run mean of volatility, indicating less strain in the SV part of the process in these models. This reduction in pressure on the SV process to generate excess kurtosis is something also observed by many others, including Bates (1996, 2000) and Bakshi *et al.* (1997) and shows the benefits of the more complex SVJ and SVJJ models.

Jumps in volatility parameters. Looking at the SVJJ model results, it is clear that the bias in long-run volatility between calls and puts has disappeared after the introduction of jumps in volatility, with this parameter having values for calls and puts that are virtually the same. The volatility of volatility, however, has increased in size over that of the SVJ model in the case of both calls and puts and is still significantly different between calls and puts (although this difference is the smallest of all the SV models). This makes it a little unclear whether the addition of jumps in volatility reduces the pressure on the SV process to generate excess kurtosis.

Time-varying skewness. The modelling of skewness in implicit parameters is dependent on the mean jump size of jumps in price and the correlation between price and volatility innovations. Throughout the sample period the size and direction of implicit skewness tended to be very irregular and episodic through time,[23] therefore looking at these parameters through time allows a more informative study

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[23] While this behaviour is not consistent with the implicit assumption of constant (or slow-changing) parameters, it is unavoidable that skewness behaves this way in Australia (skewness is generally much more consistent in the US). The inconsistent, random nature of skewness, therefore, lends itself to modelling as a random process and this is explored by Christoffersen *et al.* (2006).

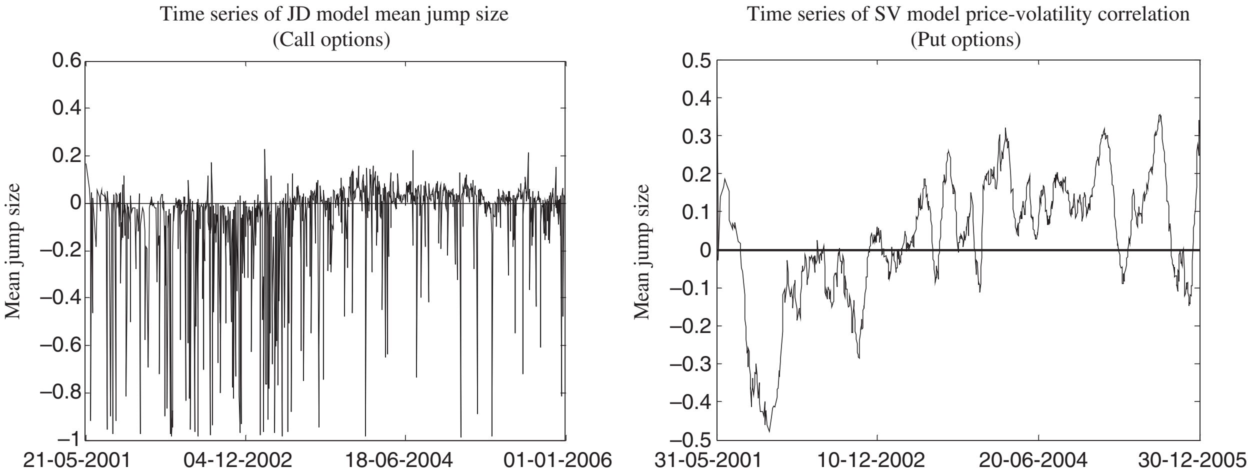

Fig. 2. Time series of model skewness
Notes: This figure shows firstly the mean jump size, μ_s, of the JD model for put options and secondly the correlation between price and volatility innovations for the SV model for put options. The correlation series has been smoothed to a monthly (based on a 21 trading day month) moving average.

of their behaviour. We look at the JD and SV models so we can see each effect independently.

Firstly, we first look at the skewness which is due to a nonzero mean jump size by looking at the JD model. Figure 2 shows the implicit mean jump size for the JD model for put options throughout the sample. It presents compelling evidence of 'crash fears' in the implicit distribution of put options, with results qualitatively similar to the period shortly before the crash of 1987 (Bates, 1991, Fig. 6C). Visible are large negative implicit jump sizes especially at and following 11 September 2001, and on 29 March 2005, the day meteorologists predicted another tsunami in South East Asia.[24] Periods of positive average jumps are also evident during the start of the bull market in 2003 and 2004. For call options (not shown) jumps follow a similar patter, but with far less large negative average jump sizes.

Secondly, we will look at skewness due to the correlation between price and volatility innovations in the SV process by looking at the SV model. The behaviour of this parameter is even more varying and inconsistent than the mean jumps size, therefore we look at it using a monthly (based on a 21 trading day month) moving average (Fig. 2).

SV model correlations seem to follow qualitatively the same path as JD model mean jump size, indicating that there is clear time-changing skewness in implicit distributions, and that it seems to be captured by both models to some degree. Call options also show the same characteristic of having fewer strongly negative correlation periods, suggesting that any 'crash fears' do not contribute significantly to the call option implicit distribution.

VI. Concluding Remarks

This article has studied four option pricing models from the class of affine models and the BS model. With the exception of the BS model, which is available in closed-form, we used tractable pricing formulae for each of the models in semi-closed form. These formulae are based on transform and inversion techniques, such as those developed by Bakshi and Madan (2000) and Duffie *et al.* (2000). This article is, to the best of authors' knowledge, the first application of an SVJJ model with independent jumps.[25] We further presented an efficient method for numerically evaluating the integrals associated with such option pricing formulae.

In studying the utility of each extra feature of an option pricing model we find that it is SV that is of first order importance in improving over the BS model. We find that the best fitting model depends on the type of option being priced (either put or call) and that the risk neutral distributions implicit in call options are substantially different to that of put options.

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[24] The tsunami never arrived and as such, the effect was very short-lived.

[25] Andersen *et al.* (2002) and Eraker *et al.* (2003) apply an SVJJ model with simultaneous, correlated jumps to the time-series of underlying prices. Eraker (2004) studies this model using options data.

We find that jumps in price are especially relevant for put options, with the best model being the SVJ model, while jumps in volatility are required to fit call options with the best choice being an SVJJ model.

Under the assumption of constant (or slow-changing) parameters the best choice is a more parsimonious model than described above, since more complex models suffer from over fitting of an implicit distribution that changes over time. In this case, the best model choice for call options is the SV model. For put options, the JD and SV models perform equally well.

The literature on option pricing and the literature on the modelling of price processes are, in recent times, beginning to merge, creating an exciting new area for research. This article, by identifying the best option pricing model (and hence distributional assumption for the underlying), paves the way for further research into the consistency of the implicit distribution with the objective distribution (with consideration of risk premiums) and the study of the forecasting efficiency of the implicit distribution.

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